technische universität dortmund

control and cyberphysical systems

Implicit predictors in regularized data-driven predictive control (DPC)

Manuel Klädtke and Moritz Schulze Darup

Motivation

- Instead of a model, DPC uses collected trajectory data directly for predictions
- DPC predictions need regularization in presence of noise or nonlinearities
- Choice and tuning of regularization drastically impact DPC performance

Contributions

- We introduce implicit predictors, which characterize the predictive behavior that DPC implicitly attributes to the system
- Their structure mainly depends on regularization and (output) constraints
- ➢ Choice and tuning of regularization is comparable to traditional model selection

Implicit predictors for DPC with 2-norm regularization

Implicit predictors: $\hat{\mathbf{y}}_{\mathrm{DPC}}(\boldsymbol{\xi}, \mathbf{u}_f)$

$$\min_{\mathbf{u}_f, \mathbf{y}_f, \boldsymbol{a}} \|\mathbf{y}_f\|_{\mathcal{Q}}^2 + \|\mathbf{u}_f\|_{\mathcal{R}}^2 + h(\boldsymbol{a}) \text{ s.t.}$$

$$\begin{pmatrix} \boldsymbol{\xi} \\ \mathbf{u}_f \\ \mathbf{y}_f \end{pmatrix} = \boldsymbol{\mathcal{D}}\boldsymbol{a} = \begin{pmatrix} \boldsymbol{W}_p \\ \boldsymbol{U}_f \\ \boldsymbol{Y}_f \end{pmatrix} \boldsymbol{a}, \quad (\mathbf{u}_f, \mathbf{y}_f) \in \mathcal{U} \times \mathcal{Y}$$

- Deterministic and LTI: Rank deficiency of $\boldsymbol{\mathcal{D}}$ implies predictor (choose $h(\boldsymbol{a}) = 0$)
- Noise: $\boldsymbol{\mathcal{D}}$ (likely) has full row rank but $h(\boldsymbol{a})$ implies predictor via optimality
- ➢ Parametrically solve for predictor via $\hat{\mathbf{y}}_{\mathrm{DPC}}(\boldsymbol{\xi}, \mathbf{u}_f) = \arg\min_{\mathbf{y}_f} \min_{\boldsymbol{a}} \ldots \text{ s.t. } \ldots$
- ➢ Adding constraint $\mathbf{y}_f = \hat{\mathbf{y}}_{\mathrm{DPC}}(\boldsymbol{\xi}, \mathbf{u}_f)$ would not change solutions

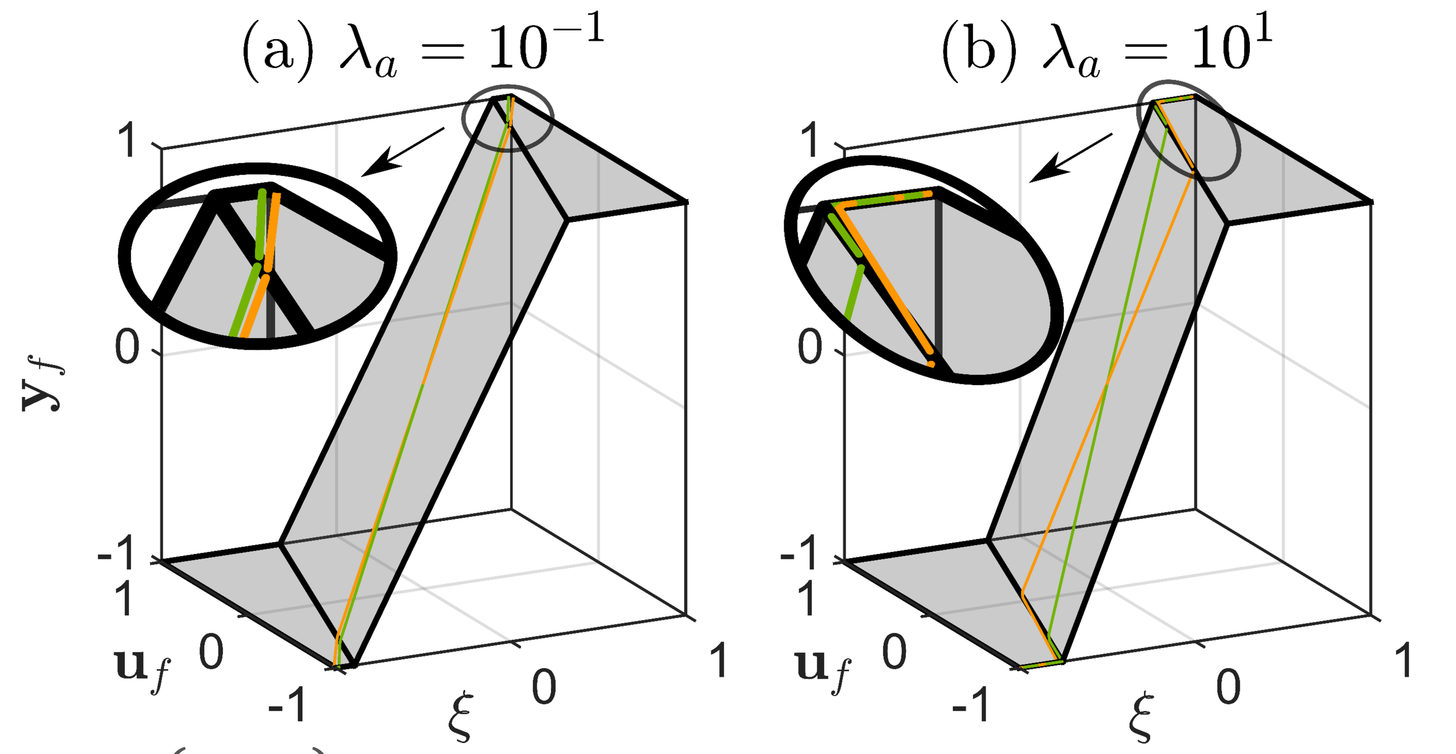
Unconstrained with 2-norm

- Popular choice: $h(\boldsymbol{a}) = \lambda_a \|\boldsymbol{a}\|_2^2$
- ➢ $\hat{\mathbf{y}}_{\mathrm{DPC}}(\boldsymbol{\xi}, \mathbf{u}_f)$ is linear and "tilts" between...
  - linear least squares fit $\hat{\mathbf{y}}_{\mathrm{SPC}}(\boldsymbol{\xi}, \mathbf{u}_f)$
  - unregularized solution $\arg\min_{\mathbf{y}_f} \|\mathbf{y}_f\|_{\mathcal{Q}}^2 = \mathbf{0}$

  depending on λ_a

Adding constraints

- Affected by $\mathbf{y}_f \in \mathcal{Y}$ but not $\mathbf{u}_f \in \mathcal{U}$
- ➢ Intuition: Changing predictor such that $\hat{\mathbf{y}}_{\mathrm{DPC}}(\boldsymbol{\xi}, \mathbf{u}_f) \in \mathcal{Y}$ holds is always more optimal than infeasibility



Grey: $\hat{\mathbf{y}}_{\mathrm{DPC}}(\boldsymbol{\xi}, \mathbf{u}_f)$, Orange/Green: parametric DPC solutions
"Plateaus" forming due to $\hat{\mathbf{y}}_{\mathrm{DPC}}(\boldsymbol{\xi}, \mathbf{u}_f) \in \mathcal{Y}$

Outlook

- Analyze other regularizations, e.g., $\|\boldsymbol{a}\|_p$
- Add common DPC modifications
- Explain popular experiments from new viewpoint of implicit predictors
- Give practical choice/tuning guidelines

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