technische universität dortmund

control and cyberphysical systems

Neural networks for predictive control

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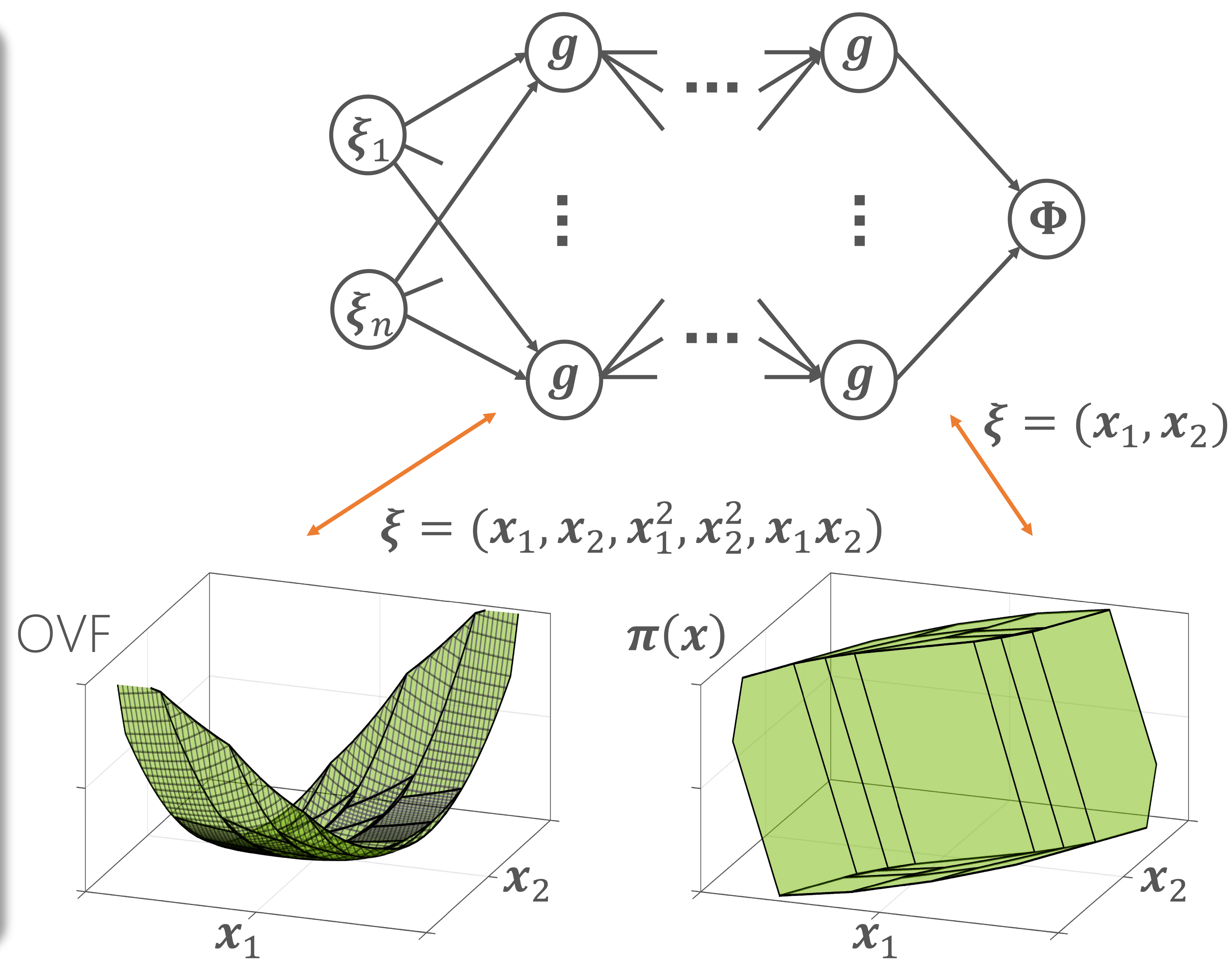
Motivation

- NN can approximate a large class of functions with arbitrary accuracy
- Good topologies leading to $\boldsymbol{\Phi}(\boldsymbol{x}) \approx \boldsymbol{\pi}(\boldsymbol{x})$ are often unknown

- Use common PWA structure of NN and the control law $\boldsymbol{\pi}(\boldsymbol{x})$ of linear MPC to derive design guidelines
- For $\boldsymbol{\xi} = (x_1, x_2, x_1^2, x_2^2, x_1 x_2)$, the approach is extendable to PWQ optimal value functions



Stability and performance verification

Mixed-integer formulation of MPC

- Computation of Lipschitz constants for the control law $\boldsymbol{\pi}(\boldsymbol{x})$
- Reduce online complexity of MPC by removing constraints that are never active

$$\max_{\boldsymbol{x},\boldsymbol{\delta}} \varphi(\boldsymbol{x}, \boldsymbol{\delta})$$
$$\text{s.t. } \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{\delta}) \leq \boldsymbol{0}, \boldsymbol{x} \in \mathcal{X}, \boldsymbol{\delta} \in \{0,1\}^N$$

Mixed-integer formulation of NN

- Compute local gain of NN
- Find min/max of the output of an NN
- Convex outer approximations of save sets

Verification of approximate MPC

- Stability is typically not preserved for approximations of the control law
- Stability of the approximate control law can by verified based on the error:
  $$\boldsymbol{e}(\boldsymbol{x}) \coloneqq \boldsymbol{\Phi}(\boldsymbol{x}) - \boldsymbol{\pi}(\boldsymbol{x})$$
- For ReLU and maxout NN, $\boldsymbol{\Phi}(\boldsymbol{x})$ can be modelled using binary variables
- Modelling of active constraints of quadratic programs by binary variables

- $\boldsymbol{e}(\boldsymbol{x})$ is the solution of a mixed-integer LP

Challenges

- Efficient training of NN for approximating MPC
- Combination with methods from reinforcement learning
- Extension to problems with high dimensional state space

Based on contributions to the 22nd IFAC WC 2023, 20th ECC 2022, and 60th CDC 2021